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Acoustoelectric effect in a semiconductor superlattice

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Abstract. The acoustoelectric effect in a semiconductor superlattice has been studied for an acoustic wave whose wavelength $\lambda = 2\pi/q$ is smaller than the mean free path l of the electrons (where $ql \gg 1$). Unlike the homogeneous bulk material where the Weinreich relation is independent of the wavenumber q in the superlattice we observe a dependence on q, i.e. spatial dispersion. In the presence of an applied constant field E a threshold value was obtained where the acoustoelectric current changes direction.

1. Introduction

When an acoustic wave is absorbed by a conductor, the transfer of momentum from the acoustic wave to the conduction electron may give rise to a current usually called the acoustoelectric current J^{ac} or, in the case of an open circuit, a constant electric field E^{ac} [1, 2]. Both the phenomenological and the microscopic theory of this effect have been comprehensively studied for bulk materials [3–10].

In this paper, we shall study this effect in a semiconductor superlattice (SL) because of the immense interest currently associated with this novel material. It is also our opinion that the study of SLs could complement the already extensively studied electrical and optical properties [11] and thus enhance understanding of the properties of the material. Experimental evidence of the dependence of the acoustoelectric effect on the SL parameters has been reported [12]. It was indicated in the article that, when the sign of the acoustoelectric current is known the major carrier in the SL can be determined. In view of this we have decided to study this effect theoretically and to elaborate more on it.

It will be shown that the presence of the minibands in the SL will result in a non-linear dependence of the acoustoelectric current and the ratio J^{ac}/Γ_z, where Γ_z is the absorption coefficient, on the wavenumber q. Also, in the presence of an applied constant field E a threshold value E_0 is obtained where the acoustoelectric current changes direction.

This paper is organized as follows. In section 2 we outline the theory and conditions necessary to solve the problem. In section 3 we discuss the results and in section 4 we draw some conclusions.

2. Theory

We shall follow the approach developed in [13] and calculate the acoustoelectric current in the SL. It is known that, when the wavelength $\lambda = 2\pi/q$ of the sound is considerably

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shorter than the electron mean free path l (where $ql \gg 1$), the sound wave can be treated as a packet of coherent phonons (monochromatic phonons) having a δ-function distribution

$$N(k) = \frac{(2\pi)^3}{\hbar\omega_q s}\phi\delta(k-q) \qquad \hbar = 1 \tag{1}$$

where k is the current phonon wavevector, $\hbar$ is Planck's constant divided by 2π, ϕ is the sound flux density, and ω_q and s are the frequency and the group velocity of sound wave with the wavevector q.

It is assumed that the sound wave and the applied constant electric field E propagate along the z axis of the SL. The problem will be solved in the quasi-classical case, i.e. $2\Delta \gg \tau^{-1} (\hbar = 1), eED \ll 2\Delta$ (d is the period of the SL, 2Δ is the width of the lowest-energy miniband and e is the electron charge). The density of the acoustoelectric current can then be written in the form [13]

$$j^{ac} = \frac{2e}{(2\pi)^3}\int U^{ac}\psi_i(p)\,\mathrm{d}^3p \tag{2}$$

where

$$U^{ac} = \frac{2\pi\phi}{\omega_q s}|G_{p-q,p}|^2[f(\epsilon_{p-q}) - f(\epsilon_p)]\delta(\epsilon_{p-q} - \epsilon_p + \omega_q)$$
$$+ |G_{p+q,p}|^2[f(\epsilon_{p+q}) - f(\epsilon_p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q) \tag{3}$$

and $\Psi_i(p)$ is the solution of the Boltzmann kinetic equation in the absence of a magnetic field. If we introduce a new term $p' = p - q$ in the first term of the integrals in equation (2) and take account of the fact that

$$|G_{p'p}|^2 = |G_{pp'}|^2$$

we can express equation (2) in the form

$$j^{ac} = -\frac{e\phi}{2\pi^2 s\omega_q}\int |G(p,q)|^2[f(\epsilon_{p+q}) - f(\epsilon_p)][\Psi_i(p+q) - \Psi_i(p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q)\,\mathrm{d}^3p \tag{4}$$

where the vector $\Psi_i(p)$ as indicated in [13] is the mean free path $l_i(p)$.

Thus the acoustoelectric current in equation (4) in the direction of SL axis becomes

$$j_z^{ac} = \frac{e\phi}{2\pi^2 s\omega_q}\int |G(p,q)|^2[f(\epsilon_{p+q}) - f(\epsilon_p)][l_z(p+q) - l_z(p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q). \tag{5}$$

where $f(\epsilon_p)$ is the distribution function, p is the momentum of the electrons, $G(p_z, q)$ is the matrix element of the electron–phonon interaction and for $qd \ll 1$ it is given as

$$|G(p_z q)|^2 = \frac{|\Lambda|^2 q^2}{2\sigma\omega_q} \tag{6}$$

where Λ is the deformation potential constant. σ is the density of the SL. As indicated in [13], in the τ approximation and further when τ is taken to be constant,

$$l_z = \tau v_z \tag{7}$$

where

$$v_z = \frac{\partial \epsilon}{\partial p_z}.$$

Inserting (6) and (7) into (5), we obtain

$$j_z^{ac} = -\frac{e\phi|\Lambda|^2 q^2 \tau}{4\pi^2 \sigma s \omega_q^2} \int [f(\epsilon_{p+q}) - f(\epsilon_p)][v_z(p+q) - v_z(p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q)\, \mathrm{d}^3 p. \qquad (8)$$

The distribution function in the presence of the applied constant field E is obtained by solving the Boltzmann equation in the τ approximation. This is given by

$$f(p) = \int_0^\infty \frac{\mathrm{d}t}{\tau} \exp(-t/\tau) f_0(p - eEt). \qquad (9)$$

Here

$$f_0(p) = \frac{\pi d n}{m T I_0(\Delta/T)} \exp(-\epsilon_p/T) \qquad (10)$$

where n is the electron density, T is the temperature in energy units and $I_0(x)$ is the Bessel function of order zero with an imaginary argument.

The energy $\epsilon(p)$ of the SL in the lowest miniband is given using the usual notation by

$$\epsilon(p) = \frac{p_\perp^2}{2m} + \Delta[1 - \cos(p_z d)]. \qquad (11)$$

Hence

$$v_z(p) = \Delta d \sin(p_z d). \qquad (12)$$

Substituting (9), (11) and (12) onto (8) and solving for a non-degenerate electron gas we obtain for the acoustoelectric current

$$j_z^{ac} = -\frac{e\phi|\Lambda|^2 q^2 n d \tau \Theta(1-b^2)}{\sigma s \omega_q^2} \int_0^\infty \frac{\mathrm{d}t}{\tau} \exp(-t'/\tau) \left\{ \sinh\left(\frac{\omega_q}{2T}\cos(eEdt')\right) \right.$$
$$\times \sinh\left[\frac{\Delta}{T}\cos(qd/2)\cos(eEdt')\sqrt{1-b^2}\right] - \frac{\Delta}{T}\sqrt{1-b^2}\sin(eEdt')\sin(qd/2)$$
$$\left. \times \cosh\left(\frac{\omega_q}{2T}\cos(eEdt')\right)\cosh\left(\frac{\Delta}{T}\sqrt{1-b^2}\cos(qd/2)\cos(eEdt')\right) \right\} \qquad (13)$$

where Θ is the Heaviside step function, $b = \omega_q/2\Delta\sin(qd/2)$

3. Results and discussion

We shall solve (13) for two particular cases.

(i) In the absence of the applied constant field ($E = 0$), from equation (13) we obtain

$$j_z^{ac} = -\frac{e\phi|\Delta|^2 q^2 \tau n d \Theta(1-b^2)}{\sigma s \omega_q^2} \sinh\left(\frac{\omega_q}{2T}\right) \sinh\left[\frac{\Delta}{T}\cos(qd/2)\sqrt{1-b^2}\right]. \quad (14)$$

It can be observed from (14) that, when $\omega_q \gg 2\Delta\sin(qd/2)$, $j_z^{ac} = 0$ and also the dependence of j_z^{ac} on q is strongly non-linear. This effect has been observed in [14] for the situation where

$$G(p_X, q) = ib[\sin(p_X + q)d - \sin(p_X d)]$$

and where b is the derivative of the resonance integral with respect to the interatomic distance and $i = \sqrt{-1}$. The ratio j_z/Γ_z where Γ_z is calculated under the same conditions as in [15], is given by

$$j_z/\Gamma_z = -\frac{2e\phi\tau d\Delta\sin(qd/2)}{\omega_q}\sqrt{1-b^2}\tanh\left[\frac{\Delta}{T}\cos(qd/2)\sqrt{1-b^2}\right]. \quad (15)$$

Hence, we observe that j_z/Γ_z depends on the wavenumber q, i.e. it has a spatial dispersion. This behaviour is unlike the homogeneous semiconductor (bulk material) which is independent of q.

It is worth noting that, at $\Delta \gg T$, when the SL is behaving as a homogeneous semiconductor, j_z/Γ_z as given in equation (15) satisfies the Weinreich relation

$$j_z/\Gamma_z = -\frac{e\phi\tau}{ms}. \quad (16)$$

As stated in [14] the change in the sign of the acoustoelectric current can be attributed to the fact that the main contribution of the acoustoelectric current at $qd \simeq 2\pi$ is from electrons near the top of the miniband, i.e. by electrons with a negative effective mass.

(ii) In a weak constant electric field, $eEd\tau \ll 1$, $\omega_q \ll T$, from (13) we obtain

$$j_z^{ac} = j_0^{ac}\left\{1 - \frac{eEd\tau}{\omega_q}\sqrt{[2\sin(qd/2)]^2 - \omega_q^2}\coth\left[\frac{\Delta}{T}\cos(qd/2)\sqrt{1-b^2}\right]\right\}. \quad (17)$$

From (17) it is observed that at

$$E > E_0 = \omega_q \frac{\tanh[(\Delta/T)\cos(qd/2)\sqrt{1-b^2}]}{e\tau d\sqrt{[2\sin(qd/2)]^2 - \omega_q^2}} \quad (18)$$

the acoustoelectric current changes sign. The value E_0 can be interpreted as a threshold field. E_0 is a function of the SL parameters d and Δ, temperature T, frequency ω_q and the wavenumber q. Thus the SL parameters can be chosen such that we find a value for E_0 where the acoustoelectric current will change direction. For example, at $\Delta/t \gg 1$, $\Delta = 0.1$ eV, $d = 5\times10^{-7}$ cm, $\tau = 10^{-12}$ s, $s = 5\times10^5$ cm s^{-1} and $\omega_q = 10^{10}$ s^{-1}. For these values we obtain the threshold field $E_0 = 8.65$ V cm^{-1} which is small and can be observed practically.

4. Conclusion

Acoustoelectric current in a SL has been studied theoretically. It has been observed that, unlike the homogeneous semiconductor, in a SL the acoustoelectric current is non-linear and the ratio j_z/Γ_z depends on q. In the limiting case when $\Delta \gg T$ we obtain the Weinreich relation for bulk materials.

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